FREE VIBRATION ANALYSIS OF CELLULAR CURVED PLATE STRUCTURES

Sadjad A. Hemzah
M.Sc.in Civil Engineering,
University of AL-Qadisiya

Munaf A. Abdulkadhem
M.Sc.in Civil Engineering,
University of AL-Qadisiya

Abstract

A free vibration analysis of a curved plate cellular structure is presented. The flexibility matrices of both curved and straight members were used in the analysis; also a lumped mass matrix having the same number of equations as the consistent mass matrix was adopted herein. ***Jacobi's*** iterative method was used to solve the Eigenvalue problems. The general purpose finite element program ***(MSC/NASTRAN)*** was used in the comparisons of the vibration analysis. The maximum deference in the results compared with ***(MSC/NASTRAN)*** are about (15%) for the fifth Eigenvalue and it decrease until (1%) for the first Eigenvalue.

تحليل الاهتزاز الحر للمنشاءات المنحنية الخلوية

سجاد عامر حمزة
ماجستير هندسة مدنية
جامعة القادسية – كلية الهندسة

مناف عبد المهدي عبد الكاظم
ماجستير هندسة مدنية
جامعة القادسية – كلية الهندسة

الخلاصة:-

تم تقديم تحليل الاهتزاز الحر للمشاءات المنحنية أفقيا. لقد استخدمت مصفوفات المرونة لكلا الأعضاء المنحنية والمستقيمة وتم تبني مصفوفة الكتلة المركزة لجميع الأعضاء. ايضا عملت طريقة Jacobi's iterative method لحل مسائل الـ(Eigenvalue).كما وتم مقارنة النتائج مع النتائج المستحصلة من برنامج العناصر المحددة (***MSC/NASTRAN***) وكان الفرق بالنتائج عند مقارنتها مع (***MSC/NASTRAN***) يساوي تقريبا (15%) في الـ Eigenvalue الخامسة ويتناقص إلى أن يصل إلى تقريبا (1%) في الـ Eigenvalue الأولى في معظم الامثلة.

Introduction

There is a growing interest in the development of reliable beam models which can be used to analyze thin-walled structures. Thin-walled beams have been widely used in many civil, mechanical and aerospace engineering applications, both in their stand-alone forms and as stiffeners for plate and shell structures. The thin walled beam theory was established by ***Vlasov*** (1961) (12) and ***Timoshenko*** and ***Gere*** (1961) (10), also the coupled bending-torsional vibration of beam has been investigated in recent years by ***Firberg*** (1993) (4) and ***Banerje*** (1996) (3). ***Trahair*** and ***Pi*** (1997) (11) summarized a series of investigations on bending-torsional vibration of beams. A consistent finite element formulation for the free vibration was presented by ***Kim***(2000) (7). Natural frequencies of small amplitude vibrations for thin walled bars of open sections were studied by ***Wekezer*** (1987) (13), who utilized the finite element method .His thin-walled finite element considered a special case of a membrane shell with internal constraints. The stiffness and consistent mass matrices were too complex to be derived in a closed form. ***Wekezer***(1989) (14) also developed a general consistent mass matrix for thin-walled bars with constant cross section for the analysis of small amplitude vibrations of such problems . ***Al-Azawi*** (2000) (1) used the displacement field for determining the stiffness matrix of both curved and straight members. Warping deformation was included in the analysis; also the lumped mass matrix was used for representing the total mass matrix of the cellular plate structure curved in plan for analyzing the free vibration of such structure. For this purpose, the subspace iterative method was used to solve the well-known characteristics problem (Eigen problem) (1).

$$K\,\Phi = \Delta\, M\,\Phi \qquad (1)$$

where ***K*** and ***M*** are the stiffness and mass matrices respectively. The Eigenvalues Δ and eigenvectors Φ are the free vibration frequencies and their corresponding mode shapes. ***Husain et al*** (5) studied the free flexural vibration of rectangular thin plates with variable thickness of cross section by using a simplified computational procedure. The discretization of the problem was carried out by means of finite differences.

In the present study, the stability matrix was used to evaluate the stiffness matrix of curved member. The exact stiffness matrix of straight member was used for the evaluation of stiffness of straight member. Regarding the mass matrix of both curved and straight members, a lumped mass matrix having the same number of equations as the consistent mass matrix will often give more accurate natural frequencies was used herein.

Idealization of Structures:-

The cellular plate structures consist of curved beams having constant section properties along the beams length, and straight beams with linearly varying width (varying properties) along the beams length as shown in Figure(2). The structures will be idealized as a grillage system, and the linearly varying width members was taken as straight members having constant section properties by assuming that b= (b1+b2 /2) (i.e. the average width of the members). This assumption makes the calculations of stiffness matrix of straight members simple and easy for calculations.

Flexibility Stiffness Matrix of Cellular Curved Beams.

To determine the flexibility stiffness matrix of a curved member, it is necessary to investigate the internal forces which act along the curve beam, as shown in Figure (1). These forces are [2]:

$$V_x = F_{xi}$$
$$M_y = -F_{xi} R \sin\theta + M_{yi} \cos\theta - M_{ti} \sin\theta \qquad (2)$$
$$M_t = -F_{xi} R(1-\cos\theta) + M_{yi} \sin\theta + M_{ti} \cos\theta$$

where :
V_x = is the shear force at any section.
M_y = is the mending moment at any section.
M_t = is the twisting moment at any section.

The nodal displacement (deflection, rotation and twisting angle) can be evaluated by applying Castigliano's second theorem, where:

$$\delta xi = \frac{\partial U}{\partial Fxi} \qquad \theta yi = \frac{\partial U}{\partial Myi} \qquad \psi i = \frac{\partial U}{\partial Mti} \qquad (3)$$

The flexibility matrix is derived from the principle of strain energy. The following deformations are considered:

Bending Moment Deformation

The strain energy expression for the deformations resulting from the bending moment can be expressed for prismatic member as:

$$U_b = \frac{R}{2EI}\int_0^{\beta} M_y^2 \, d\theta \qquad (4)$$

After introducing the moment equation M_y from Equation (2) and integrating with respect to θ the strain energy due to the bending moment deformations may be expressed as:

$$U_b = \frac{R}{2EI}\left[M_{yi}^2\left(\frac{\beta+sc}{2}\right) - M_{yi}M_{ti}s^2 + M_{ti}^2\left(\frac{\beta-sc}{2}\right) - M_{yi}F_{xi}Rs^2 + 2F_{xi}M_{ti}R\left(\frac{\beta-sc}{2}\right) + F_{xi}^2R^2\left(\frac{\beta-sc}{2}\right)\right] \qquad (5)$$

where:
s : sin(θ)
c : cos(θ)

Then, the flexibility matrix of a curve beam element due to bending deformations can be expressed as:

$$[F_{by}] = \frac{R^3}{EI}\begin{bmatrix} \frac{(\beta-sc)}{2} & -\frac{1}{2R}s^2 & \frac{(\beta-sc)}{2R} \\ & \frac{(\beta+sc)}{2R^2} & -\frac{1}{2R^2}s^2 \\ & & \frac{(\beta-sc)}{2R^2} \end{bmatrix} \qquad (6)$$

Torsional deformations

The strain energy expression for the deformations resulting from the torsional moment can be expressed as:

$$U_t = \frac{R}{2GJ}\int_0^{\beta} M_t^2 d\theta \qquad (7)$$

After introducing the moment equation $\boldsymbol{M_t}$ from Equation (2) and integrating with respect to $\boldsymbol{\theta}$ and applying Castigliano's second theorem, then the flexibility matrix of a curve beam element due to torsional deformations can be expressed as:

$$[F_{mt}] = \frac{1}{RGJ}\begin{bmatrix} \frac{sc-4s+3\beta}{2} & -\frac{(1-c)^2}{2R} & \frac{(2s-\beta-sc)}{2R} \\ & \frac{(\beta-sc)}{2R^2} & -\frac{s^2}{2R^2} \\ & & \frac{(\beta+sc)}{2R^2} \end{bmatrix} \qquad (8)$$

Shear deformations

The strain energy expression for direct shear deformation may be expressed for a curve beam element as:

$$U_s = \frac{R}{2EA}\int_0^{\beta} V_t^2 d\theta \qquad (9)$$

where E is the modulus of elasticity and A is the cross sectional area. After integrating and differentiating with respect to the force $\boldsymbol{F_{xi}}$, the displacement $\boldsymbol{\delta_{xi}}$ can be written as:

$$\delta_{xi} = \left(\frac{R}{EA}\right)F_{xi} \qquad (10)$$

Then the total flexibility matrix of the curved beam element is:

$$[F] = [F_{by}] + [F_{mt}] + [F_{sx}] \qquad (11)$$

The stiffness matrix **[*K*],** which relates the forces at node (i), can be obtained from the inversion of flexibility matrix constructed in equation (11). The coefficient of the stiffness matrix **[K_{jj}]** are the same as those of the stiffness matrix **[K_{ii}],** expect that the off-diagonal terms have opposite signs.

The other stiffness matrix coefficients of **[K_{ij}]** and **[K_{ji}]** are obtained by using static equilibrium:

$$[Kji] = [Kij]^T = [T][Kii] \qquad (12)$$

where [T] is the transformation matrix of forces form node (i) to node (j):

$$[T]=\begin{bmatrix} -1 & 0 & 0 \\ R\sin\beta & -\cos\beta & \sin\beta \\ R(1-\cos\beta) & -\sin\beta & -\cos\beta \end{bmatrix} \qquad (13)$$

By this operation the complete stiffness matrix of order (6 × 6) is developed.

Stiffness Matrix of Straight Beams.

The following stiffness matrix was used to calculate the stiffness matrix of straight member [(6)], which represents the exact stiffness of straight members. This stiffness matrix is widely used in the analysis and design of many types of structures. The properties of element in the structure were considered to be the average of these at the end of the beam as follows:

$$I=(I_1+I_2)/2$$
$$J=(J_1+J_2)/2 \qquad (14)$$

$$K_S=\begin{bmatrix} \frac{12EI}{L^3(1+\phi)} & & & & & \\ \frac{6EI}{L^2(1+\phi)} & \frac{EI(4+\phi)}{L(1+\phi)} & & sym & & \\ 0 & 0 & \frac{GJ}{L} & & & \\ \frac{-12EI}{L^3(1+\phi)} & -\frac{6EI}{L^2(1+\phi)} & 0 & \frac{12EI}{L^3(1+\phi)} & & \\ \frac{6EI}{L^2(1+\phi)} & \frac{EI(2+\phi)}{L\,(1+\phi)} & & \frac{-6EI}{L^2(1+\phi)} & \frac{EI(4+\phi)}{L\,(1+\phi)} & \\ & & \frac{-GJ}{L} & & & \frac{GJ}{L} \end{bmatrix} \qquad (15)$$

Formulation of The Mass Matrix:

In addition to the stiffness of cellular structures a property matrix in the dynamic behavior is the inertia or mass, which represented by a mass matrix. There are two possible types of expressions for mass matrices, lumped mass and consistent mass. In the lumped mass, the mass is assumed to be concentrated at the node while the consistent mass is derived from Hamelton's principle (which is a variational method utilizing the minimum total potential energy)

The mass and inertia of the length, area or volume tributary to a particular node are considered to be concentrated at the node. This method greatly facilitates matrix calculation [(2)]. As mentioned previously the mass matrix of both curved and straight members were calculated as a lumped mass matrix having the same number of equations as the consistent mass matrix[(2)]. The following lumped mass matrix was used for both types of beams:

$$Ms = \frac{\rho L}{2}\begin{bmatrix} A & & & & & \\ 0 & I & & sym & & \\ 0 & 0 & J & & & \\ 0 & 0 & 0 & A & & \\ 0 & 0 & 0 & 0 & I & \\ 0 & 0 & 0 & 0 & 0 & J \end{bmatrix} \qquad (16)$$

where:
A = Area of the section
I = Moment of inertia of the section
J = Torsional constant of the section

Programming of Section Properties

A computer program (***SECPRO***) is coded in FORTRAN 90 language was written by ***Hemzah*** (2003) [(9)] to evaluate the section properties of any cellular section having (n) cells and with different dimensions. These properties are:
I_x: moment of inertia of the member.
J : Torsional constant which of the member .
The torsional constant was calculated from the following equation:

$$2A_i = \left(-\psi_{i-1}\int_w \frac{ds}{t} + \psi_i \oint_i \frac{ds}{t} - \psi_{i+1}\int_w \frac{ds}{t} \right) \qquad (17)$$

where

$\psi_i = \dfrac{q_{svi}}{G\theta'}$ for the cell i

$\psi_{i-1} = \dfrac{q_{svi}}{G\theta'}$ for the adjacent left cell

$\psi_{i+1} = \dfrac{q_{svi}}{G\theta'}$ for the adjacent right cell

for a cellular section, the torsional constant (***Ji***) to each cell is:

$$J_i = 2.A_i.\psi_i \qquad (18)$$

also, the moment of inertia of the member was calculated by considering the beam has ***I*** section.

Solution Method

The simplest Eigen problem encountered in structural engineering has the standard form:

$$K\Phi = \Lambda\Phi \qquad (19)$$

Matrix ***K*** is the stiffness matrix of the structural assemblage, which of order (n×n) and that K is positive definite. There are n eigenvalues and corresponding eigenvector satisfying Equation (17). The ***ith*** Eigenpair is denoted as (Λi, Φi) where eigen values are ordered according to their magnitudes:

$$0 \leq \Lambda_1 \leq \Lambda_2 \leq \Lambda_3 \leq \Lambda_4 \ldots\ldots\ldots \leq \Lambda_{n-1} \leq \Lambda_n \quad (20)$$

In vibration mode superposition analysis, the generalized eigenproblem is given as:

$$K\Phi = M\,\Lambda\Phi \quad (21)$$

The matrix M is the global mass matrix of the assemblage, which can be a banded consistent mass matrix, or a diagonal lumped mass matrix. Several approximate and exact techniques are available for the analysis of the free vibration problems. For the cellular deck structures ***Jacobi's*** iterative method was used for this purpose. A computer program coded in ***FORTRAN90*** language was recoded as subroutine in a program named as (***JACOBI.F90***). All the Eigenvalues and the associated Eigen vectors were found for each example. The only five lowest values of Eigenvalues were taken to be compared with the available results.

Numerical Examples

For the solution of free vibration of cellular plate structures, two types of plated structure were analyzed, these are:

Cellular Plate Structure

In this example four types of cellular plate structure curved in plan were analyzed. In all of these examples the section and material properties were fixed and only the support condition was changed as follows (as shown in Figures (3, 4, 5, and 6)):
Simply supported at all edges.
Fixed support at radial edges.
Simply supported at radial edges.
Cantilever edge.

The section properties and dimensions of cellular plate structure are shown in Figure (7); the density of the steel material was taking to be 7850 kg/m3.

Ribbed Plate Structure :

Also here four types of ribbed plate structure curved in plan were analyzed. The properties of these structures are shown Figure (8). Again different types of support conditions similar to those used in cellular plate structure were used in the analysis, these are:
Simply supported at all edges.
Fixed support at radial edges.
Simply supported at radial edges.
Cantilever edge.

For both types of plated structure the results obtained from the present method give a good agreement with those obtained from package program ***(MSC/NASTRAN)*** [8]**.** The maximum deference in the results compared with ***(MSC/NASTRAN)*** are about (15%) for the fifth Eigenvalue and it decrease until (1%) for the first Eigenvalue.

Conclusion

In order to assess the efficiency and accuracy of the present study for free vibration analysis of steel cellular plate structures curved in plan, a number of examples are analyzed. The results of the Eigenvalues for the first five modes of vibration by the present study with the corresponding finite element values are presented in **Tables (1,2,…8).** It can be seen that the Eigenvalues resulting from the finite element method and those from the present study are in close agreement than other methods. It is worth mentioning that the results of the present study are also close to each other. For the first two modes, the difference between the finite element and the present study values can be attributed mainly to the difficulty of predicting accurately the shear lag effects in dynamic problems of thin-walled structures. The Eigenvalues by the present method are slightly larger than the values by the finite element (for all examples). This phenomenon again indicates that the present idealization gives less stiffness when compared with the finite elements.

Also, the tapering effect (in the flange width) was not considered here, because of the idealization of tapered members as straight members having the average widths, this effect may be studied for the free vibration analysis of such cellular members.

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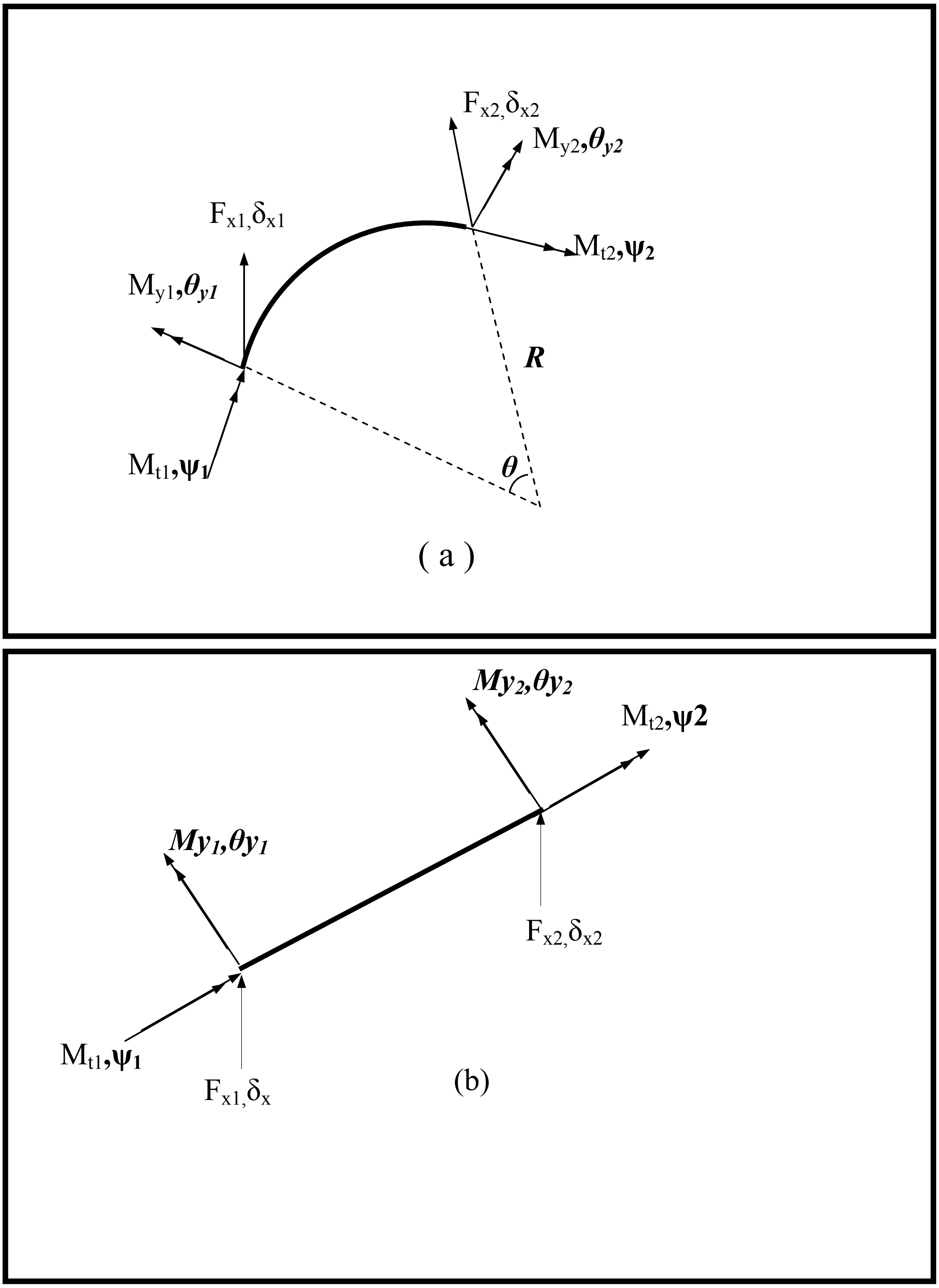


Fig.(1): Sign convention of grillage members
(a) Curved member
(b) Straight member

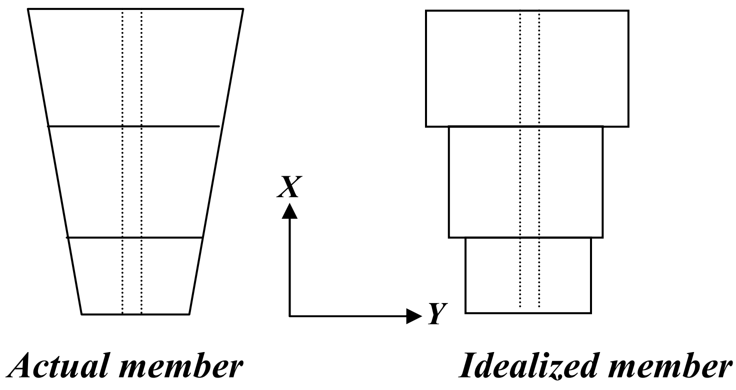


Fig. (2): Actual and idealized radial straight member

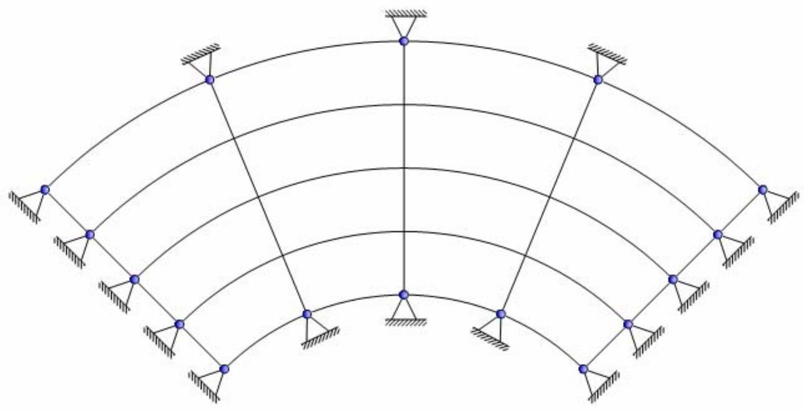


Fig.(3) Cellular structure simply supported all around

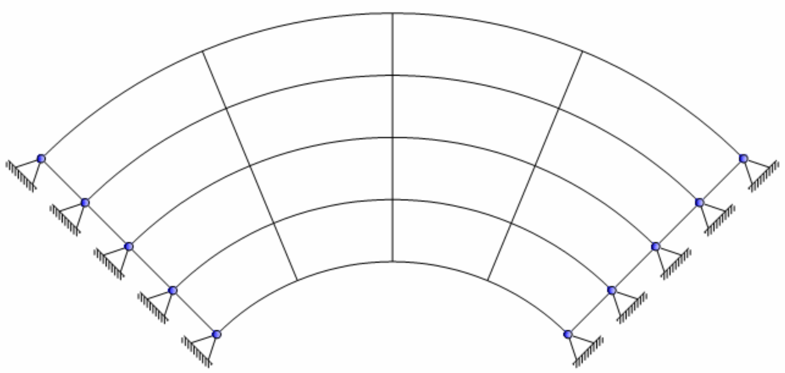


Fig. (4) Cellular structure simply supported at radial edge

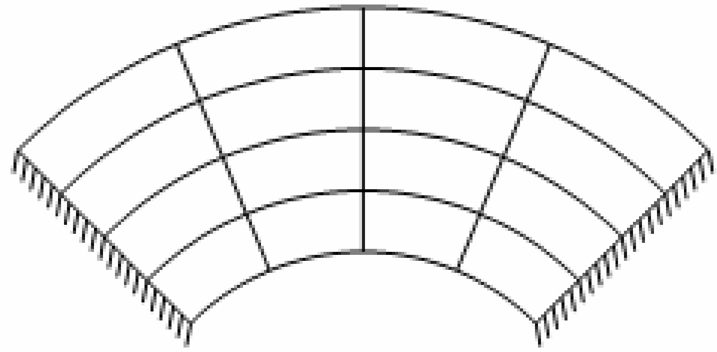


Fig.(5) Cellular structure fixed at radial edge

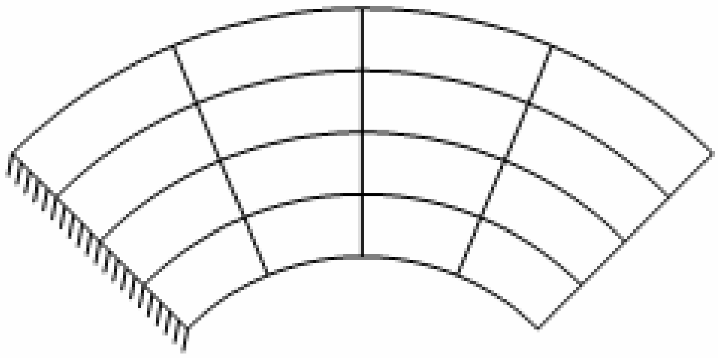
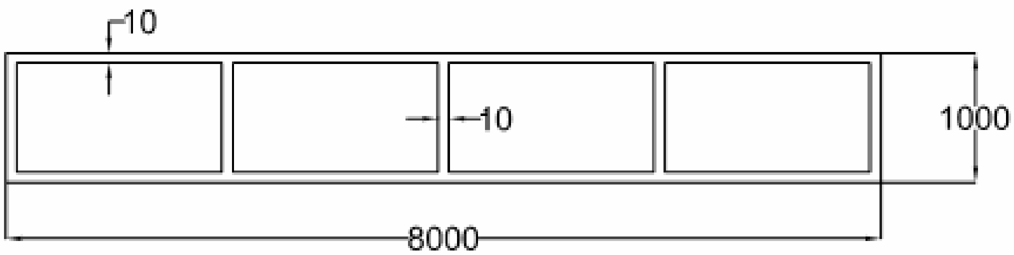
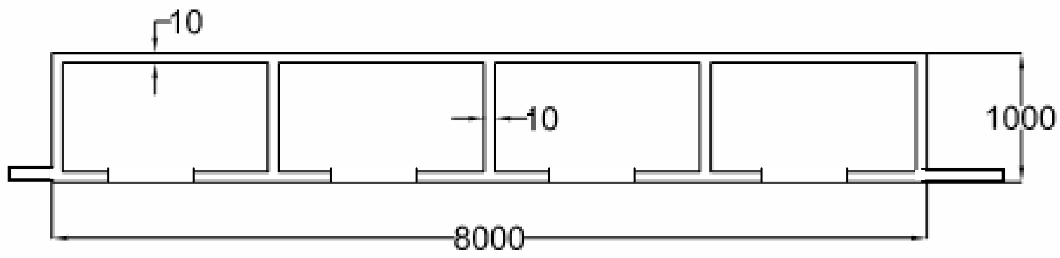


Fig.(6) Cantilever cellular structure



R= 8000
t= 10 for all plates
v=0.3
All dimensions are in mm
ρ= 7850 Kg/m^3

Fig.(7) Section properties and dimensions of cellular plate structure



R= 8000
t= 10 for all plates
v=0.3
All dimensions are in mm
***ρ*=** 7850 Kg/m^3

Section a-a

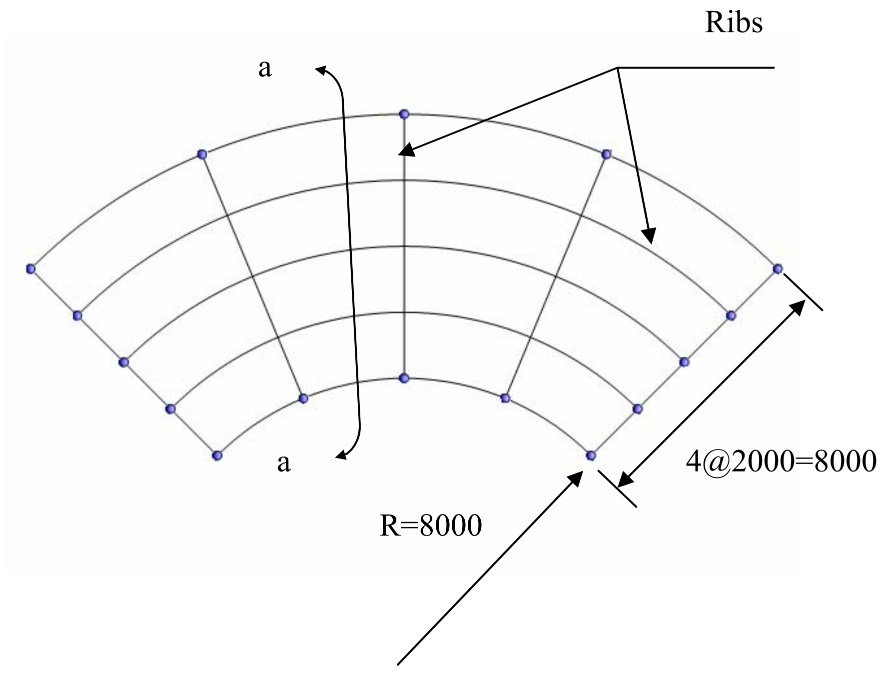


Fig.(8) Section properties and dimensions of ribbed plate structure

Table (1): Eigenvalues of the simply supported at all edge cellular structure(Hz).

Case	Mode No.	Grillage analogy[1]		*Current Study*	F.E.M
		Without warping	Without warping		
Simply supported at all edges	1	0.573	0.576	0.579	0.583
	2	0.820	0.840	0.888	0.890
	3	1.110	1.095	1.130	1.221
	4	1.332	1.147	1.428	1.631
	5	1.467	1.275	1.532	1.789

Table (2): Eigenvalues of the fixed at radial edge cellular structure(Hz).

Case	Mode No.	Grillage analogy[1]		*Current Study*	F.E.M
		Without warping	Without warping		
Fixed at radial edge	1	0.165	0.169	0.169	0.170
	2	0.447	0.461	0.471	0.491
	3	0.490	0.481	0.499	0.516
	4	0.923	0.859	0.930	1.023
	5	0.992	0.891	1.102	1.440

Table (3): Eigenvalues of the simply supported at radial edge cellular structure(Hz).

Case	Mode No.	Grillage analogy[1]		*Current Study*	F.E.M
		Without warping	Without warping		
Simply supported at radial edge	1	0.165	0.169	0.169	0.170
	2	0. 447	0.461	0.481	0.491
	3	0. 490	0.480	0.499	0.516
	4	0. 923	0.859	0.930	1.023
	5	0.992	0.891	1.102	1.440

Table (4): Eigenvalues of the cantilever edge cellular structure(Hz).

Case	Mode No.	Grillage analogy[1]		*Current Study*	F.E.M
		Without warping	Without warping		
Cantilever edge	1	0.159	0.154	0.160	0.162
	2	0.244	0.231	0.248	0.258
	3	0.289	0.257	0.292	0.399
	4	0.642	0.540	0.687	0.751
	5	0.699	0.590	0.796	0.923

Table (5): Eigenvalues of the simply supported at all edge ribbed structure (Hz).

Case	Mode No.	F.E.M	*Current Study*
Simply supported at all edge	1	0.235	0.239
	2	0.344	0.350
	3	0.386	0.394
	4	0.390	0.399
	5	0.392	0.401

Table (6): Eigenvalues of the fixed at radial edge ribbed structure (Hz)..

Case	Mode No.	F.E.M	*Current Study*
Fixed at radial edge	1	0.270	0.275
	2	0.295	0.299
	3	0.333	0.341
	4	0.382	0.390
	5	0.421	0.432

Table (7): Eigenvalues of the simply supported at radial edge ribbed structure (Hz).

Case	Mode No.	F.E.M	*Current Study*
Simply supported at radial edge	1	0.270	0.275
	2	0.295	0.298
	3	0.333	0.342
	4	0.382	0.392
	5	0.422	0.434

Table (8): Eigenvalues of the cantilever edge ribbed structure (Hz).

Case	Mode No.	F.E.M	*Current Study*
Simply supported at radial edge	1	0.015	0.016
	2	0.035	0.036
	3	0.039	0.041
	4	0.044	0.050
	5	0.049	0.055